FUZZY BAIRE RESOLVABLE and FUZZY BAIRE IRRESOLVABLE SPACES

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27.05.2019

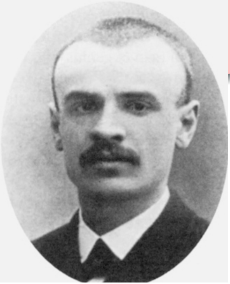
Introduction

The notion of fuzzy sets
as a new approach to a mathematical representation of vagueness in everyday language, was introduced by L.A.Zadeh [13] in classical paper in the year 1965.

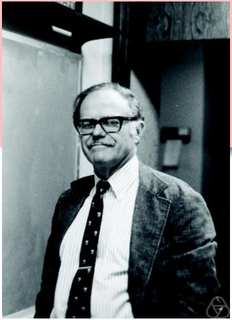
In 1968, C.L.Chang[4] introduced the concept of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

Introduction cont...

In 1899, Rene Louis
Baire [3] introduced the concepts of first category and second category sets in his doctoral thesis.



Introduction cont...



E. Hewitt [6] introduced
the concepts of resolvability and irresolvability
in topological spaces. In the recent years,
a considerable amount of research has been done on
various types of fuzzy sets in fuzzy topological spaces.

Introduction cont...

The concepts of resolvability, irresolvability in fuzzy setting were introduced and studied by G. Thangaraj and G. Balasubramanian [7].

The purpose of this presentation is to introduce and study fuzzy Baire dense sets in fuzzy topological spaces. A new class of fuzzy topological spaces, called fuzzy Baire resolvable space is introduced by means of fuzzy Baire dense sets. Several characterizations of fuzzy Baire resolvable spaces, are established. The conditions for fuzzy topological spaces to become fuzzy Baire resolvable spaces, fuzzy resolvable spaces, are obtained by means of fuzzy Baire dense sets. Several examples are given to illustrate the concepts introduced in this presentation.

Definition 2.1[4]

Let (X,T) be a topological space and λ be any fuzzy set in (X, T).The interior and the closure of λ are define as follows:

(i) . $\text{int}(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$.

(ii) . $\text{cl}(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.1[1]

Let λ be any fuzzy set in a fuzzy topological space (X, T). Then $1 - cl(\lambda) = int(1 - \lambda)$ and $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.2[1]

A fuzzy set λ in a fuzzy topological space (X, T) is called a

(i) . fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X, T)[7].

(ii) . fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu \leq cl(\lambda)$. That is, $intcl(\lambda) = 0$, in (X, T)[7].

(iii) . fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category. [7]

(iv) . fuzzy residual set, if $1 - \lambda$ is a fuzzy first category set in (X, T)[9].

(v) . fuzzy G_δ-set in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where λ_i for $i \in I$[4].

(vi) . fuzzy F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$[4].

(vii) . fuzzy Baire set in (X, T) if $\lambda = \mu \wedge \delta$ where,μ is a fuzzy open and δ is a fuzzy residual set in (X, T)[11].

Definition 2.3[1]

A fuzzy topological space (X, T) is called a

(i) . fuzzy Baire space if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T)[9].

(ii) . fuzzy submaximal space if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, then $\lambda \in T$ in (X, T)[4].

(iii) . fuzzy resolvable space if there exists a fuzzy dense set λ in (X, T) such that $1 - \lambda$ is also a fuzzy dense set in (X, T). Otherwise (X, T) is called a fuzzy irresolvable space [8].

(iv) . fuzzy P-space if $\wedge_{i=1}^{\infty}(\lambda_i) = \lambda$ where (λ)'s are fuzzy open sets in (X, T). That is., every non-zero fuzzy G_δ-set is fuzzy open in (X, T)[1].

Theorem 2.1[2]

(a) . The closure of a fuzzy open set is a fuzzy regular closed set, and

(b) . the interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.2[10]

If λ is a fuzzy σ-boundary set in a fuzzy topological space (X, T), λ is a fuzzy F_σ-set in (X, T).

Theorem 2.3[9]

If the fuzzy topological space (X, T) is a fuzzy Baire space, then no non-zero fuzzy open set is a fuzzy first category set in (X, T).

Theorem 2.4[1]

If λ is a fuzzy nowhere dense set in a fuzzy submaximal space (X, T), then λ is a fuzzy closed set in (X, T).

Theorem 2.5[9]

Let (X, T) be a fuzzy topological space. Then the following are equivalent:

(i) . (X, T) is a fuzzy Baire space.

(ii) .$int(\lambda) = 0$, for every fuzzy first category set λ in (X, T).

(iii) .$cl(\mu) = 1$, for every fuzzy residual set μ in (X, T).

Theorem 2.6[10]

If λ is a fuzzy Baire set in a fuzzy submaximal and fuzzy P-space (X, T), then λ is a fuzzy open set in (X, T).

Theorem 2.7[1]

If λ is a fuzzy G_δ-set in a fuzzy Baire and fuzzy P-space (X, T), then λ is a fuzzy second category set in (X, T).

Theorem 2.8[1]

If λ is a fuzzy F_σ-set in a fuzzy P-space (X, T), then λ is a fuzzy closed set in (X, T).

Theorem 2.9[10]

If λ is a fuzzy Baire set in a fuzzy topological space (X, T), then there exists a fuzzy Baire set β in (X, T) such that $\beta \leq \lambda$.

Theorem 2.10[10]

If λ is a fuzzy Baire set in a fuzzy Baire space (X, T), then there exists a fuzzy regular open set γ in (X, T) such that $1 - \lambda \geq \gamma$.

Theorem 2.11[10]

If λ is a fuzzy Baire set in a fuzzy Baire space (X, T), then λ is not a fuzzy dense set in (X, T).

3. FUZZY BAIRE DENSE SETS

Definition 3.1

Let (X, T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy Baire dense set in (X, T), if for a non-zero fuzzy open set μ in (X, T), $\lambda \wedge \mu$ is a fuzzy second category set in (X, T).

Example (3.1)

Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, μ, α, β, γ, δ, and σ defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0{,}4; \lambda(b) = 0{,}6; \lambda(c) = 0{,}5$.

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0{,}5; \mu(b) = 0{,}5; \mu(c) = 0{,}7$.

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0{,}4; \alpha(b) = 0{,}5; \alpha(c) = 0{,}2$.

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0{,}3; \beta(b) = 0{,}4, \beta(c) = 0{,}3$;

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0{,}5; \gamma(b) = 0{,}5; \gamma(c) = 0{,}2$;

$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0{,}3; \delta(b) = 0{,}5; \delta(c) = 0{,}5$;

$\sigma : X \to [0, 1]$ is defined as $\sigma(a) = 0{,}5; \sigma(b) = 0{,}6; \sigma(c) = 0{,}4$;

Example (3.1 cont...)

Clearly, $T = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ is a fuzzy topology on X. On computation, $int[cl(1-\mu) = 0$, $int[cl(1-\lambda \vee \mu) = 0$, $int[cl(\alpha)] = 0$, $int[cl(\beta) = 0$, $int[cl(\gamma) = 0$, implies that $1-\mu, 1-\lambda \vee \mu, \alpha, \beta, \gamma$ are fuzzy nowhere dense set in (X, T).

Now, $(1-\mu) = (1-\lambda \vee \mu) \vee \alpha \vee \beta \vee \gamma$ implies that $1-\mu$ is a fuzzy first category set in (X, T). Since, the fuzzy sets δ and σ, on computation one can see that $\delta \wedge \lambda$, $\delta \wedge \lambda$, $\delta \wedge [\lambda \vee \mu]$, $\delta \wedge [\lambda \wedge \mu]$, $\sigma \wedge \lambda$, $\sigma \wedge \mu$, $\sigma \wedge [\lambda \vee \mu]$, $\sigma \wedge [\lambda \wedge \mu]$, are not fuzzy first category sets in (X, T), and hence they are fuzzy second category sets in (X, T). Thus, δ, and σ are fuzzy Baire dense sets in (X, T).

Remarks 3.1

A fuzzy Baire dense set need not be a fuzzy dense set in a fuzzy topological space. For, in example 3.1, δ is a fuzzy Baire dense set in (X, T) but not a fuzzy dense set in (X, T) since $cl(\delta) = 1 - [\lambda \wedge \mu]$.

Example (3.2)

Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, μ, α, β, γ, δ, ρ and σ defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0{,}3; \lambda(b) = 0{,}6; \lambda(c) = 0{,}5$.
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0{,}5; \mu(b) = 0{,}4; \mu(c) = 0{,}7$.
$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0{,}4; \alpha(b) = 0{,}5; \alpha(c) = 0{,}2$.
$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0{,}3; \beta(b) = 0{,}4, \beta(c) = 0{,}2;$
$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0{,}5; \delta(b) = 0{,}5; \delta(c) = 0{,}3;$
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0{,}5; \gamma(b) = 0{,}5; \gamma(c) = 0{,}8;$
$\sigma : X \to [0, 1]$ is defined as $\sigma(a) = 0{,}5; \sigma(b) = 0{,}7; \sigma(c) = 0{,}3;$
$\rho : X \to [0, 1]$ is defined as $\rho(a) = 0{,}5; \rho(b) = 0{,}7; \rho(c) = 0{,}3;$
Clearly, $T = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ is a fuzzy topology on X. On computation, $int[cl(1 - \mu) = 0$, $int[cl(1 - \lambda \vee \mu) = 0$, $int[cl(\alpha)] = 0$, $int[cl(\beta) = 0$, $int[cl(\delta) = 0$, $int[cl(1 - \gamma) = 0$, $int[cl(\rho) = 0$, implies that $1 - \mu, 1 - \lambda \vee \mu, \alpha, \beta, \delta, 1 - \gamma, \rho$ are fuzzy nowhere dense set in (X, T).

Example (3.2 cont...)

Now, $(1-\mu) = (1-\lambda \vee \mu) \vee \alpha \vee \beta \vee \delta \vee 1-\gamma \vee \rho$ and $\delta = (1-\lambda \vee \mu) \vee \alpha \vee \beta \vee 1-\gamma$ implies that $1-\mu$ and δ is a fuzzy first category set in (X, T). Since, the fuzzy set σ, one can see that $\sigma \wedge [\lambda \vee \mu] = 1-\mu$, shows that $\sigma \wedge [\lambda \vee \mu$ is a fuzzy first category set in (X, T) and hence σ is not a fuzzy second category set in (X, T). Thus, σ is not a fuzzy Baire dense sets in (X, T).

Proposition 3.1

If λ is a fuzzy Baire dense set in a fuzzy topological space (X, T), then for a fuzzy open set μ in (X, T), $\lambda \wedge \mu \neq \vee_{i=1}^{\infty}(\delta_i)$, where $(\delta_i)'s$ are fuzzy nowhere dense sets in (X, T).

Proposition 3.2

If λ is a fuzzy Baire dense set in a fuzzy topological space (X, T), then for a fuzzy open set μ in (X, T), $1-(\lambda \wedge \mu) \neq \wedge_{i=1}^{\infty}(\gamma_i)$, where $cl(\gamma_i) = 1$, in (X, T).

Proposition 3.3

If each fuzzy open set is a fuzzy second category set in a fuzzy topological space (X, T) and if λ is a fuzzy set defined on X such that $\lambda \geq \mu$, for a fuzzy open set μ in (X, T), then λ is a fuzzy Baire dense set in (X, T).

Proposition 3.4

If λ is a fuzzy Baire dense set in a fuzzy topological space (X, T), then there exists a fuzzy second category set δ such that $\delta \leq \lambda$ in (X, T).

Proposition 3.5

If each fuzzy second category set is a fuzzy dense set in a fuzzy topological space (X, T) and if λ is a fuzzy Baire dense set in (X, T), then λ is a fuzzy dense set in (X, T).

Proposition 3.6

If λ is a fuzzy Baire dense set in a fuzzy topological space (X, T), then for a fuzzy open set μ in (X, T), such that $\lambda \wedge \mu$ is not a fuzzy $F_\sigma-$set in (X, T).

Proposition 3.7

If λ is a fuzzy Baire dense set in a fuzzy submaximal space (X, T), then for a fuzzy open set μ in (X, T), such that $1 - \lambda \wedge \mu$ is not a fuzzy $G_\delta-$set in (X, T).

Proposition 3.8

If λ is a fuzzy set in a fuzzy Baire space (X, T) such that $\lambda \geq \mu$, for a fuzzy open set μ in (X, T), then λ is a fuzzy Baire dense set in (X, T).

Proposition 3.9

If λ is a fuzzy set in a fuzzy Baire and fuzzy P-space in (X, T) such that $\lambda \wedge \mu$ is a fuzzy G_δ-set, for a fuzzy open set μ in (X, T), then λ is a fuzzy Baire dense set in (X, T).

Proposition 3.10

If λ is a fuzzy Baire dense set in a fuzzy Baire space (X, T), then for a fuzzy open set μ in (X, T), $int(\lambda)\$int \not\leq 1 - \mu$, in (X, T).

Proposition 3.11

If λ is a fuzzy Baire dense set in a fuzzy Baire space (X, T), then for a fuzzy open set μ in (X, T),

(i) . there exists a fuzzy regular closed set δ in (X, T) such that $\delta \leq cl(\lambda \wedge \mu)$.

(ii) . $1 - (\lambda \wedge \mu)$ is not a fuzzy dense set in (X, T).

Proposition 3.12

If each fuzzy open sets are fuzzy second category sets in a fuzzy submaximal and fuzzy P-space (X, T) and if λ is a fuzzy Baire set in (X, T), then λ is a fuzzy Baire dense set in (X, T).

Proposition 3.13

If λ is a fuzzy G_δ-set in a fuzzy Baire and fuzzy P-space (X, T) such that $\lambda \leq \mu$, for a fuzzy open set μ in (X, T), then λ is a fuzzy Baire dense set in (X, T).

Proposition 3.14

If λ is a fuzzy F_σ-set in a fuzzy Baire and fuzzy P-space (X, T) such that $\lambda \geq \mu$, for a fuzzy closed set μ in (X, T), then $1 - \lambda$ is a fuzzy Baire dense in (X, T).

Proposition 3.15

If λ is a fuzzy F_σ-set in a fuzzy Baire and fuzzy P-space (X, T) such that $\lambda \geq \mu$, for a fuzzy closed set μ in (X, T), then $1 - \lambda$ is a fuzzy Baire dense set and fuzzy open set in (X, T).

Proposition 3.16

If λ is a fuzzy open set in a fuzzy Baire space (X, T), then λ is a fuzzy Baire dense set in (X, T).

Proposition 3.17

If λ is a fuzzy G_δ-set in a fuzzy Baire and fuzzy P-space (X, T), then λ is a fuzzy Baire dense set in (X, T).

Proposition 3.18

If λ is a fuzzy set in a fuzzy Baire and fuzzy P-space (X, T), such that $\lambda \wedge \mu$ is a fuzzy G_δ-set, for a fuzzy open set μ in (X, T), then $\lambda \wedge \mu$ is a fuzzy Baire dense set in (X, T).

Proposition 3.19

If λ is a fuzzy Baire dense set in a fuzzy Baire space (X, T), then

(i) . $int(\lambda) \neq 0$ in (X, T).

(ii) . $cl(1 - \lambda) \neq 1$, in (X, T).

Proposition 3.20

If fuzzy open sets are fuzzy second category sets in a fuzzy submaximal and fuzzy P-space (X, T) and if λ is a fuzzy Baire set in (X, T), then there exists a fuzzy Baire dense set β in (X, T) such that $\beta \leq \lambda$.

Proposition 3.21

If λ is a fuzzy Baire set in a fuzzy Baire space (X, T) then there exists a fuzzy Baire dense set γ in (X, T) such that $1 - \lambda \geq \gamma$.

Proposition 3.22

If λ is a fuzzy Baire set in a fuzzy Baire space (X, T) then there exists a fuzzy Baire dense set γ in (X, T) such that $1 - \lambda \geq \gamma$ and $cl(\gamma) \neq 1$.

Proposition 3.23

If λ is a fuzzy Baire set in a fuzzy Baire space (X, T) then there exists a fuzzy second category set δ such that $1 - \lambda \geq \delta$ in (X, T).

Proposition 3.24

If λ is a fuzzy Baire set in a fuzzy submaximal and fuzzy P-space (X, T) in which fuzzy open sets are fuzzy second category sets, then there exists a fuzzy second category set δ such that $\delta \leq \lambda$, in (X, T).

Proposition 3.25

If $(\lambda_i)'s$ $(i = 1 to \infty)$ are fuzzy sets in a fuzzy Baire and fuzzy P-space (X, T) such that $\lambda_i \geq \mu_i$, for fuzzy open sets μ_i in (X, T), then $\wedge_{i=1}^{\infty}(\lambda_i)$ is a fuzzy Baire dense set in (X, T).

4. FUZZY BAIRE DENSE SETS

Definition 4.1

A fuzzy topological space (X, T) is called a fuzzy Baire resolvable space if there exists a fuzzy Baire dense set λ in (X, T), such that $1 - \lambda$ is also fuzzy Baire dense set in (X, T). Otherwise (X, T) is called a fuzzy Baire irresolvable space.

Example (4.1)

Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, α, β, and γ, defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0{,}4; \lambda(b) = 0{,}7; \lambda(c) = 0{,}4$.

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0{,}9; \alpha(b) = 1; \alpha(c) = 0{,}2$.

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0{,}2; \beta(b) = 0{,}3, \beta(c) = 1;$

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 1; \gamma(b) = 0{,}7; \gamma(c) = 0{,}4;$

Example (4.1 cont...)

Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \gamma \wedge (\alpha \vee \beta), \beta \vee (\alpha \wedge \gamma), \alpha \vee (\beta \wedge \gamma), 1\}$ is a fuzzy topology on X. On computation, $1 - \alpha, 1 - \beta, 1 - \gamma, 1 - \alpha \vee \beta, 1 - \alpha \vee \gamma, 1 - \beta \vee \gamma, 1 - \alpha \wedge \beta, 1 - \alpha \wedge \gamma, 1 - \beta \wedge \gamma, 1 - \gamma \wedge (\alpha \vee \beta), 1 - \beta \vee (\alpha \wedge \gamma), 1 - \alpha \vee (\beta \wedge \gamma)$, are fuzzy nowhere dense set in (X, T).

Now,
$(1 - \alpha \wedge \beta) = 1 - \alpha \vee 1 - \beta \vee 1 - \gamma \vee 1 - \alpha \vee \beta \vee 1 - \alpha \vee \gamma \vee 1 - \beta \vee \gamma \vee 1 - \alpha \wedge \gamma \vee 1 - \beta \wedge \gamma \vee 1 - \gamma \wedge (\alpha \vee \beta) \vee 1 - \beta \vee (\alpha \wedge \gamma) \vee 1 - \alpha \vee (\beta \wedge \gamma)$, and $1 - \alpha \wedge \gamma = 1 - \alpha \vee 1 - \gamma \vee 1 - \alpha \vee \beta \vee 1 - \alpha \vee \gamma \vee 1 - \beta \vee \gamma \vee 1 - \alpha \wedge \gamma \vee 1 - \gamma \wedge (\alpha \vee \beta) \vee 1 - \beta \vee (\alpha \wedge \gamma) \vee 1 - \alpha \vee (\beta \wedge \gamma)$, implies that $1 - \alpha \wedge \beta)$ and $1 - \alpha \wedge \gamma$ are fuzzy first category set in (X, T). Since, the fuzzy sets λ and$1 - \lambda$,

Example (4.1 cont...)

on computation one can see that $\alpha \wedge \lambda, \beta \wedge \lambda, \gamma \wedge \lambda,$ $(\alpha \vee \beta) \wedge \lambda, (\alpha \vee \gamma) \wedge \lambda, (\beta \vee \gamma) \wedge \lambda,$ $(\alpha \wedge \beta) \wedge \lambda, (\alpha \wedge \gamma) \wedge \lambda,$ $(\beta \wedge \gamma) \wedge \lambda, [\gamma \wedge (\alpha \vee \beta)] \wedge \lambda,$ $[\beta \vee (\alpha \wedge \gamma)] \wedge \lambda, [\alpha \vee (\beta \wedge \gamma)] \wedge \lambda,$ $\alpha \wedge 1 - \lambda, \beta \wedge 1 - \lambda, \gamma \wedge \ 1 - \lambda,$ $(\alpha \vee \beta) \wedge 1 - \lambda, (\alpha \vee \gamma) \wedge 1 - \lambda,$ $(\beta \vee \gamma) \wedge 1 - \lambda,$ $(\alpha \wedge \beta) \wedge 1 - \lambda, (\alpha \wedge \gamma) \wedge 1 - \lambda,$ $(\beta \wedge \gamma) \wedge 1 - \lambda,$ $[\gamma \wedge (\alpha \vee \beta)] \wedge 1 - \lambda,$ $[\beta \vee (\alpha \wedge \gamma)] \wedge 1 - \lambda,$ $[\alpha \vee (\beta \wedge \gamma)] \wedge 1 - \lambda,$ are not fuzzy first category sets in (X, T), and hence they are fuzzy second category sets in (X, T). Thus, λ, and $1 - \lambda$ are fuzzy Baire dense sets in (X, T). Hence (X, T) is a fuzzy Baire resolvable space.

Proposition 4.1

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then (X, T) is a fuzzy resolvable space.

Proposition 4.2

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then there exists a fuzzy set λ in (X, T) such that $cl(\lambda) = 1$ and $int(\lambda) = 0$, in (X, T).

Proposition 4.3

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then there exists a fuzzy set λ in (X, T) such that $Bd(\lambda) = 1$ and $Bd(1 - \lambda) = 1$, in (X, T).

Proposition 4.4

If fuzzy open sets are fuzzy second category sets in a fuzzy submaximal and fuzzy P-space (X, T) and if λ and $1 - \lambda$ are fuzzy Baire sets in (X, T), then (X, T) is a fuzzy Baire resolvable space.

Proposition 4.5

If (X, T) is a fuzzy Baire resolvable space, then there exists a fuzzy Baire dense set λ in (X, T) such that $int(\lambda) \wedge \mu \neq 0$ and $int(1 - \lambda) \wedge \mu \neq 0$, for a fuzzy open set μ in (X, T).

Proposition 4.6

If there exists a fuzzy set λ in a fuzzy Baire space (X, T) such that $\lambda \geq \mu$ and $1 - \lambda \geq \mu$, for a fuzzy open set μ in (X, T), then (X, T) is a fuzzy Baire resolvable space.

Remarks 4.1

A fuzzy Baire resolvable space need not be a fuzzy resolvable space. For, consider the following example:

Example (4.3)

Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, α, β, γ and δ, defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0{,}4;\ \lambda(b) = 0{,}7;\ \lambda(c) = 0{,}4.$

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0{,}9;\ \alpha(b) = 1;\ \alpha(c) = 0{,}2.$

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0{,}2;\ \beta(b) = 0{,}3,\ \beta(c) = 1;$

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 1;\ \gamma(b) = 0{,}7;\ \gamma(c) = 0{,}4;$

$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0{,}9;\ \delta(b) = 0{,}7;\ \delta(c) = 0{,}8;$

Example (4.3 cont...)

Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \gamma \wedge (\alpha \vee \beta), \beta \vee (\alpha \wedge \gamma), \alpha \vee (\beta \wedge \gamma), 1\}$ is a fuzzy topology on X. As in example 4.1, λ and $(1 - \lambda)$ is a fuzzy Baire dense set in (X, T). Hence (X, T) is not a fuzzy Baire resolvable space. On computation, one can see that $cl(\delta) = 1$ and $cl(1 - \delta) = 1 - [\gamma \wedge (\alpha \vee \beta)] \neq 1$. There is no fuzzy dense set σ in (X, T) such that $cl(-\sigma) = 1$ and hence (X, T) is not a fuzzy resolvable space.

Remarks 4.2

A fuzzy resolvable space need not be a fuzzy Baire resolvable space. For, in example 4.2, on computation one can see that $cl(\lambda) = 1$ and $cl(-\lambda) = 1$ in (X, T) and hence (X, T) is a fuzzy resolvable space but (X, T) is not a fuzzy Baire resolvable space.

The following proposition gives a condition for a fuzzy Baire resolvable space to become a fuzzy resolvable space.

Proposition 4.7

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then (X, T) is a fuzzy resolvable space.

Proposition 4.8

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then there exists a fuzzy set λ in (X, T) such that $cl(\lambda) = 1$ and $int(\lambda) = 0$, in (X, T).

Proposition 4.9

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then there exists a fuzzy set λ in (X, T) such that $Bd(\lambda) = 1$ and $Bd(1 - \lambda) = 1$, in (X, T).

Proposition 4.10

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then (X, T) is not a fuzzy submaximal space.

Proposition 4.11

If each fuzzy second category set is a fuzzy dense set in a fuzzy Baire resolvable space (X, T), then there exists a fuzzy set λ in (X, T) such that $int[cl(\lambda)] + cl[int(\lambda)] = 1$.

Remarks 4.3

A fuzzy Baire resolvable space need not be a fuzzy Baire space. For, in example 4.1, (X, T) is a fuzzy Baire resolvable space but not a fuzzy Baire space since for the fuzzy first category set $1 - (\alpha \wedge \mu)$, $int[1 - (\alpha \wedge \mu)] = \beta \wedge \gamma \neq 0$, in (X, T).
The following proposition gives a condition for a fuzzy Baire space to become a fuzzy Baire resolvable space.

Proposition 4.12

If there exists a fuzzy clopen set in a fuzzy Baire space (X, T), then (X, T) is a fuzzy Baire resolvable space.
The following proposition gives a condition for fuzzy Baire space to become fuzzy resolvable space.

Proposition 4.13

If there exists a fuzzy clopen set in a fuzzy Baire space (X, T) in which fuzzy second category sets are fuzzy dense sets then (X, T) is a fuzzy resolvable space.

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Thank You!